**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

*Solution: μ* = 45 & *σ* = 8

As the service manager plans to have work begin on the

transmission of a customer’s

car 10 minutes after the car is dropped off.

Therefore, new *μ* = 55

The probability that the service manager meet his commitment=

use python code

q1=stats.norm.cdf(60,loc=55,scale=8)

q1= 0.7340

The probability that the service manager cannot meet his

commitment=1-q1

=0.2659

Option (B)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Solution: Mean *μ* = 38

Standard deviation *σ* =6

The probability of more employees at the processing center

are older than 44=

use python code

1-q1=stats.norm.cdf(44, loc=38,scale=6)= 0.15865

Only 15.865% employees at the processing center

are older than 44

Also, The probability of employees at the processing center

between 38 and 44=P(38<=X<=44)

=P(X<=44)-P(X<=38)

= q1=stats.norm.cdf(44, loc=38,scale=6)- q2=stats.norm.cdf(38,

loc=38,scale=6)

=0.3414

Only 34.14% employees at the processing center

are between 38 and 44.

Answer=False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution: The probability of employees at the processing

center under the age of 30= q1=stats.norm.cdf(30, loc=38,scale=6)

= 0.0912

Total no. of employee=400

The training program for employees under the age of 30 at the center would be expected to attract= (q1\*400)/100=0.36

Answer: True, The training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution:  *X1* ~ *N*(μ, σ2)

*X*2 ~ *N*(μ, σ2)

As from central limit theorem any large sum of independent identically distribution random variables are approximately normal.then 2X1 and X1+X2 are normal if n is large.

2X1~N(2 μ, 4σ2)

X1+X2~ *N*(2μ,2 σ2)

Now, 2X1-(X1+X2)~(2μ-2 μ, 4σ2-2 σ2)

The distribution remains same for every subset of similar source and tend to fall under normal distribution with slightly deviation in parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution: Mean *μ* = 100

Standard deviation *σ* =20

Two values, *a* and *b*, symmetric about the mean, such that the

probability of the random variable taking a value between them

is 0.99= use python code

= stats.norm.interval(0.99, loc=100,scale=20)

Answer:[48.48,151.51] option (D)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Solution: Let X be the profit of entire company

Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

E[X]=E[45\*(Profit1+profit2)]=45\*(5+7)=540million rupees

SD[X]=SD(Profit1+profit2)=45\*sqrt(9+16)=225 million rupees

Therefore X~ N(540,225)

Now the required range of probability 540+-1.96\*225=

=[99, 981] million rupees

1. Specify the 5th percentile of profit (in Rupees) for the company

Solution: 5 percentile of X =540-1.645\*225=169.875 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Solution: Z score associated with 0 for each of the division is=

For division 1: (0-5)/3=-1.67

For division 1: (0-7)/4=-1.75

Since, -1.75<-1.67,p(-0.75)<p(-1.67)

Therefore, for division 1 the probability of making loss is lower

Answer: For division 2 probability of making loss is larger.